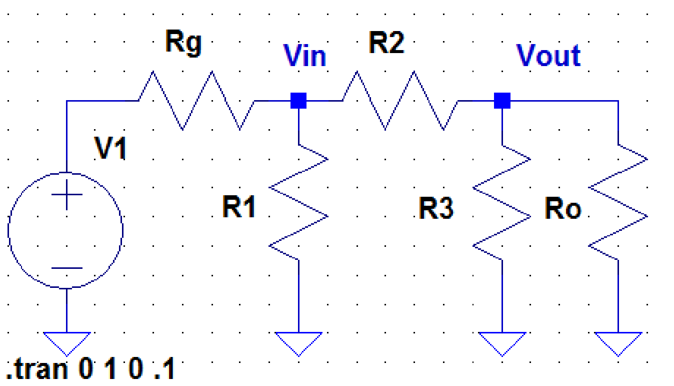
Derivation of Pi-Network Pad Resistor Formulas

The Xtal Set Society

Why on earth would one want to derive formulas; why not just use them? I like to work out the formulas because in doing so additional insights into the workings of the system/circuit are often forthcoming. I do it just because it's fun too!

Formulas for shunt and series resistors, (R1, R3) and R2, for a desired drop in voltage in a resistive pi-network pad can be derived using the circuit shown. The assumptions made are that the drive and load resistances have the same value (which does not need to be 50 ohms) and that each sees that same resistance looking into a single or multiple (attenuator) pi-network.



Given our assumptions, Rg and Rout must be the same and equal the chosen system resistance R. The input and output voltages are simply those we choose! In addition, the combination of R1, R2, R3 and Rout must equal R, as seen from the output of the generator (after Rg) and input of the pi-network at node Vin. Since Rg "sees" just a total of R at Vin, that circuit is a simple voltage divider of the open-circuit generator voltage:

(1.1) $$V_{in} = \frac{R}{R+R_g} V_g = \frac{1}{2} V_g .$$

Thus all voltages are known and it remains to determine the currents in order to obtain values for R1, R2, and R3. Let's sum the currents at each voltage node (Vin, Vout, and ground) to see if a solution falls out easily.

(1.2) $$I_{Rg} = I_{R1} + I_{R2},$$

(1.3) $$I_{R2} = I_{R3} + I_{Rout},$$

(1.4) $$I_{R1} + I_{R3} + I_{Ro} = I_{Rg}.$$

Aha! There is no need to solve three equations for three unknowns, assuming by symmetry (and experience) that R1 must equal R3. Hence we can solve for R1 using equation 1.4 alone as follows:

(1.5) $I_{Rg} - I_{Ro} = I_{R1} + I_{R3,}$

Substituting with known voltages, resistances, and equivalent resistances,

(1.6) $\frac{V_g - V_{in}}{R_g} - \frac{V_o}{R_o} = \frac{V_{in}}{R_1} + \frac{V_o}{R_1}$, and rearranging, $R_1 = R\frac{(V_{in} + V_o)}{(V_{in} - V_o)}$.

Using (1.2), we can solve for R2,

(1.7) $I_{R2} = I_{Rg} - I_{R1}$,

Again, substituting with known voltages and resistances,

(1.8) $\frac{V_{in} - V_o}{R_2} = \frac{V_{in}}{R} - \frac{V_{in}}{R_1}$, so rearranging, $\mathrm{R}_2 = \frac{(V_{in} - V_o)}{V_o}\frac{RR_1}{(R_1 - R)}$.

Check Example:

To check these results, let's design a -10 dBv pad for a 50 ohm system. Given a 1 volt input, the output would be exp(-10/20) = 0.316V. Hence,

(1.9) $R_1, R_3 = 50\frac{(1+0.306)}{(1-0.316)} = 96.2.$

(1.10) $R_2 = \frac{(1-0.316)}{0.316}\frac{50(96.2)}{(96.2-50)} = 71.21.$

These values were used to simulate the above circuit using LTspice, a circuit simulation program. V1 was given a value of 2 V; hence, Vin was 1 volt, and the output obtained was 0.316 volts.

Some Insights: Equations 1.6 and 1.8 provide some insights regarding operation of the pad. Given 1.6, R1 & R3 will vary from over 1,000 to 50 ohms as Vo varies from near Vin to zero. Given 1.8, R2 will vary from near 0 to several 100 ohms as Vo varies from near Vin to zero. *In other words, R1 & R3 trend opposite to R2 as Vout varies.* This matches intuition. As R2 is increased, R3 is decreased; thus less voltage is divided out to the load. As R2 is decreased, R3 is increased (while the load R is held constant); thus providing more voltage at the output.